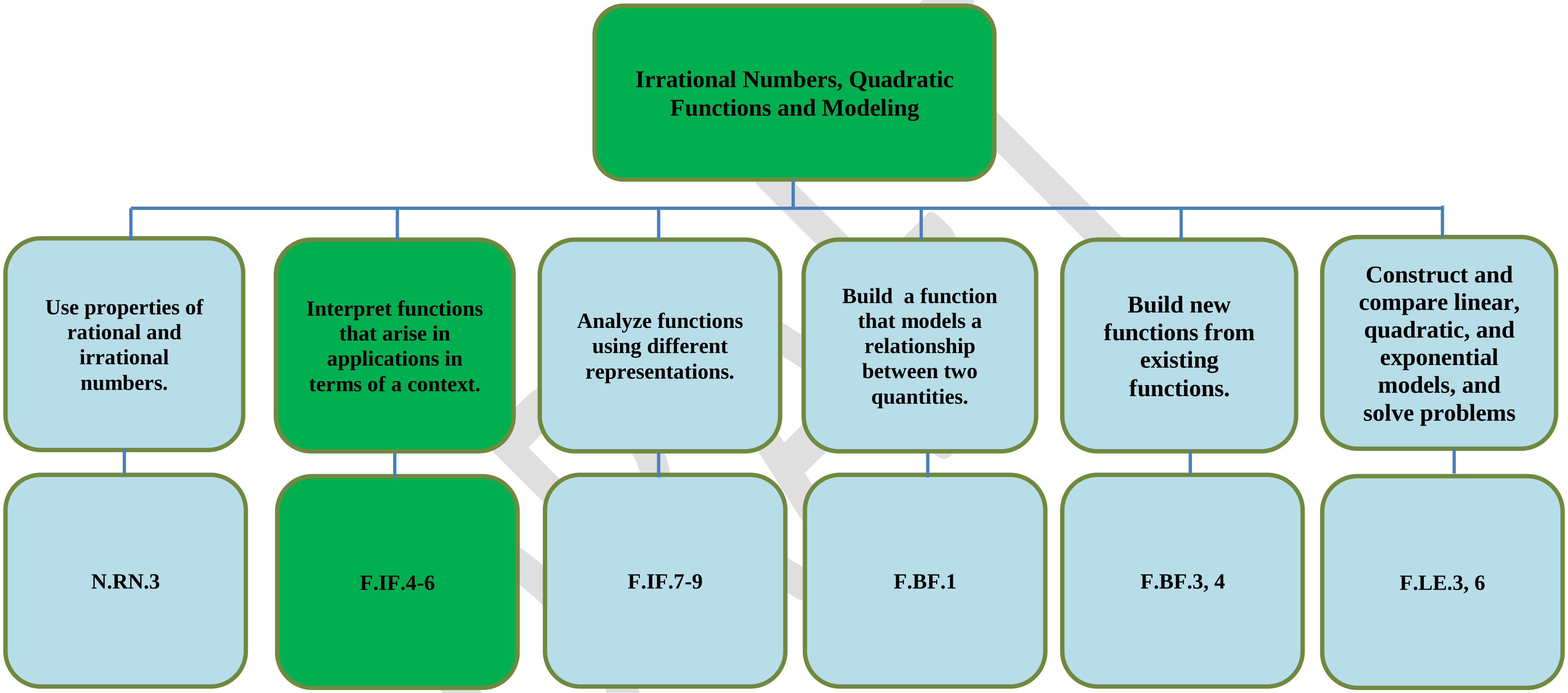
Los Angeles Unified School District
Alternative Accelerated CC Math 8/ Algebra 1

Unit 5



GRADE 7 Super Accelerated – UNIT 5
Quadratics Functions and Modeling

Critical Area:
In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions— absolute value, step, and those that are piecewise-defined.

Rationale: In designing this unit for the Grade 7 Super Accelerated Course, the same alignment and order of the standards for Quadratics was chosen as the Grade 8 Accelerated Course to support students learning progressions in this area. No additional 8^{th} grade standards were required to support student learning in this critical area. However, the 8^{th} grade Geometry standards were moved to the Grade 8 Super Accelerated course as support for pre-requisites needed in that course and to support the Super Accelerated Program as a whole. Geometry is not a critical area for the Grade 7 Super Accelerated Course.

CLUSTERS	COMMON CORE STATE STANDARDS
(s)Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area.*	**N.RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
(m)Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2*	**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
(m)Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise defined functions with linear,*	**F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.	c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions s0uch as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay*. **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function* and an algebraic expression for another, say which has the larger maximum.
(m)Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic relationship.*	**F.BF.1** Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *b.* Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
(*s*)Build new functions from existing functions. *For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x2, x>0.*	**F.BF.3** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions s0uch as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay*. **F.BF.4** Find inverse functions a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

(*s*)Construct and compare linear, quadratic, and exponential models and solve problems. *Compare linear and exponential growth to quadratic growth.*	**F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **F.LE.6** **Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.** (CA Standard Algebra I-23.0)
MATHEMATICAL PRACTICES	
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the arguments of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. **6. Attend to precision.** **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematical Practices 1, 2, 3, 4, 6, and 7 in this unit.

LEARNING PROGRESSIONS

http://ime.math.arizona.edu/progressions/#committee.

CDE Progress to Algebra K-8 www.cde.ca.gov/be/cc/cd/documents/updateditem12catt3.doc

Interactive Wire Diagram for prerequisite standards
http://www.curtiscenter.math.ucla.edu/MapApp/prg_map.html

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
Mathematical relationships can be presented graphically, in tables, or in verbal descriptions and the meaning of features in each representation can be interpreted in terms of the situation. Quadratic, linear or exponential function can be modeled, and the situation can be used in context to specify the domain and range as it relates to the understanding of real-world application of algebra concepts.	1) How does each element of the domain correspond to exactly one element of the range? 2) How would you relate and interpret features of relationships represented in a graph, table, and verbal description? 3) How can you represent the same function algebraically in different forms and interpret these differences in terms of the graph or context? 4) What differences are there in the parameters of linear, exponential, and quadratic expressions?	Completing the square Domain Exponential functions Extreme values Factoring Function Intercepts Interval Irrational number Linear function

The connection between the graph of the equation = ( ) and the function itself can be made, and the coordinates of any point on the graph represent an input and output, expressed as (, ( )). Translation between the tabular, graphical, and symbolic representations of a function can be explored between these representations and the situation’s context. Key characteristics of functions can be identified, and function language and notation to analyze and compare functions can be used. The zeros and roots of quadratic function can be solved by factoring or completing the square. Equivalent forms of linear, exponential and quadratic functions can be created to analyze and compare functions and features of functions (e.g. rates of change in specified intervals). The same function algebraically can be represented in different forms and the differences can be interpreted in terms of the graph or context. The sum or product of two rational numbers is rational can be explained, by arguing that the sum of two fractions with integer numerator and denominator is also a fraction of the same type.	5) How would you model physical problems with linear, exponential and quadratic functions and what role would their parameters play in modeling? 6) How can you find the zeros and roots of a quadratic function? 7) How do the graphs of mathematical models and data help us better understand the world in which we live? 8) How would you explain the product or sum of rational and irrational numbers?	Maxima Maximum Minima Parameter Range Rational number Relative maximum Relative minimum Root Quantitative relationship Quadratic function Symmetry Zeros

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT

Mathematics Assessment Project – MARS Task Function and Everyday Situations - F.IF.7-8 http://map.mathshell.org/materials/download.php?fileid=1259 **Illustrative Mathematics** Influenza Epidemic – F.IF.4: http://www.illustrativemathematics.org/illustrations/637 Warming and Cooling – F.IF.4: http://www.illustrativemathematics.org/illustrations/639 How is the weather – F.IF.4: http://www.illustrativemathematics.org/illustrations/649 Logistic Growth Model, Explicit Version – F.IF.4: http://www.illustrativemathematics.org/illustrations/804 The Canoe Trip, Variation 1 – F.IF.4-5: http://www.illustrativemathematics.org/illustrations/386 The High School Gym – F.IF.6b: http://www.illustrativemathematics.org/illustrations/577 Temperature Change –F.IF.6: http://www.illustrativemathematics.org/illustrations/1500	Facilitate a discussion with students that would help them represent functions with graphs and identify key features in the graph. Create or use already created activity where students would match different functions with their graphs, tables, and description. Engage students in graphing linear, exponential, and quadratic functions in order for them to develop fluency and the ability to graph them by hand. Help students to develop their idea of modeling physical problems with linear, exponential, and quadratic functions by looking at practical application of linear, quadratic, and exponential situations; such as stock market and investment, compound and simple interests, rocket trajectory, and speed of cars. Provide students the opportunity to compare linear, quadratic, and exponential functions, represented in different ways (table, graph, or situation) in writing using graphic organizers; such as T-chart or Venn diagram.	Smarter Balanced: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm PARCC Sample Assessments: http://www.parcconline.org/samples/mathematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_HSAlgIIMathIIITemperatureChange_081913_Final_0. pdf

Which Function? - F.IF.8a: http://www.illustrativemathematics.org/illustrations/640 Throwing Baseballs – F.IF.9 and F.IF.4: http://www.illustrativemathematics.org/illustrations/1279		

LANGUAGE GOALS

Students will relate and interpret orally and in writing using complex sentences the meaning and features of relationships arising from a situation – whether presented graphically, in tabular form, and/or as verbal descriptions.

Students will explain (orally and in writing) how to model a situation with a quadratic, linear or exponential function, and will be able to use the situation's context to specify the domain and range.

Students will write how to translate between the tabular, graphical, and symbolic representations of a function, and between these representations and the situation's context.

Students will identify and orally explain key characteristics of functions using the function language and notation to analyze and compare functions.

PERFORMANCE TASKS

Mathematics Assessment Project – MARS Task

- Functions and Everyday Situations – F.IF.4- 9, F.BF.3, F.LE.3: http://map.mathshell.org/materials/download.php?fileid=1259

Illumination Mathematics

- Average Cost – F.IF.B.4-5 http://www.illustrativemathematics.org/illustrations/387

Noyce Foundation – Inside Mathematics

- Sorting Functions – F.IF.4, 7a, 7c, 8a, F.LE.2 http://insidemathematics.org/common-core-math-tasks/high-school/HS-F-2008%20Sorting%20Functions.pdf

DIFFERENTIATION📖

FRONT LOADING	ACCELERATION	INTERVENTION
Prerequisites: Understanding and use the formal mathematical language of functions. Provide students an opportunity to compare two	Provide the students several opportunities to collect data to model different situations related to linear, quadratic, exponential functions, and trigonometric functions. Have students complete a project such as:	Have students evaluate different functions (linear, quadratics, and exponential) for a given variable.

functions (quadratic and exponential), represented in different ways (table, graph, or situation).	The half-life of caffeine is 6 hours. In other words, after consuming some caffeine, half of that caffeine is still present in the body after 6 hours. The amount of caffeine in the body at the end of any given time interval is A=Pd-kt where P is the amount of caffeine present in the body at the beginning of the time interval, t is the length of the time interval, and k is the decay constant. For one day from the time you wake up to the time you go to bed, keep a record of the time and the amount consumed of any beverage that contains caffeine. Research how much caffeine is in each type of drink you consumed. Calculate the amount of caffeine in your body when you went to bed that night. Compare your results with your classmates. Use your calculations and the results of others to make a conjecture about the time of day you should consume your last caffeinated beverage if you want to have less than 20 milligrams in your body when you go to sleep. What time should you consume your last caffeinated beverage if you want to have no caffeine in your body when you go to sleep? (CORD Algebra 2: Learning in Context, 2008.) Which Function? - F.IF.8a http://www.illustrativemathematics.org/illustrations/640 This activity is a nice analysis that involves a real understanding of what the equation of a translated parabola looks like.	Then engage the students in identifying appropriate domain for the functions. Help students take the "function machine" that they learned in the earlier grades and turn it into a deeper understanding of relating the situation, table, and rule (formula) of functions. The goal here is to help students make the interrventions.

[1] Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

[2] **Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.**

References:

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